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How do non-performing loans affect productivity? Evidence from Tunisian banks using a parametric hyperbolic distance function

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Abstract:

As solving the problem of the high level of non-performing loans presents a key factor in strengthening the soundness of the banking system, Tunisian government has considered the reduction of the nonperforming loans burden of a paramount importance. In this paper, we construct a new total factor productivity (TFP) index using a parametric hyperbolic distance function, which simultaneously credits for an expansion in economic outputs (loans and others earning assets) along with contractions undesirable output (nonperforming loans). This new TFP index provides more fruitful and meaningful economic decomposition. In addition to the first three parts (technical change, efficiency change and scale effect change), our index offers one more part related to undesirable output reflecting its effect on productivity. Subsequently, this index is employed to evaluate the TFP change for 10 listed

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Tunisian banks during 1992–2014. The results indicate that over the studied period, the banking sector experienced a progress in term of Efficiency change as well as a technical change by 0.5%, and 0.3%, respectively. However, these improvements were offset by regress in term of scale effect change and NPLs effect change. Thus, over the entire period (1992-2014), there was no significant change in the average TFP. Public banks are found to have been more successful than the private ones in capturing benefits from changes in technology and efficiency. In addition, the scale changes and undesirable output effects are found to be problematic for the private and public banks indicating that they do not operate at an optimal scale and do not efficiently manage their risk.

Keywords: bank Efficiency, Productivity Change, Hyperbolic distance functions, Undesirable output, NPLs, Tunisian banks

1. Introduction

Over the past two decades, banking and financial systems have undergone dramatic changes and developments worldwide. On one side, banking deregulation, financial integration and merger-acquisition contribute to the profound transformation of banking operational environment. On the other side, driven by the technological innovation, banks are able to create a range of new products and reduce costs in providing financial services. Inspired by these developments, a large body of efficiency and productivity studies was carried out in order to inform regulators and practitioners of banking sector performance, help governors review banking and financial regulation, and assist bank managers to assess and supervise their managerial ability. In the literature of performance evaluation studies, a substantial number of researchers focused on technical efficiency, cost efficiency and profit efficiency using either non-parametric or/and parametric frontier methodologies. Another strand of academic studies focused on the productivity measurement and its decomposition using either nonparametric or/and parametric frontier approaches.

Analyzing the productivity of banking systems is interesting from a policy perspective because an increase in productivity is expected to induce lower prices, enhance service quality, and improve resource allocation of the entire economy.

However, it’s worth noting that the productivity alters as a result of the differences in the production technology, the efficiency of the production process, and the scale of operations. Hence, a change in productivity can be decomposed into several sources: technical efficiency change, scale efficiency change and technological change. Technical efficiency change measures the capacity of a bank to enhance its production position relative to the production frontier from one period to another. Scale efficiency change measures changes in the scale of this bank's operations relative to the most productive scale over time. Meanwhile, technological change captures the shift in the production frontier from one period to another, reflecting the improvement or the deterioration in the performance of best-practice banks.

Our study extends the traditional parametric Malmquist productivity index to a new index that takes into account undesirable outputs (NPLs). So, by using the estimated hyperbolic distance function, the Diewert's (1976) Lemme of quadratic identity and the general approach described in Orea (2002), it was possible to define a new TFP index, which provides a more significant and rewarding economic decomposition. The new TFP index offers an additional source reflecting the undesirable outputs effect on productivity. This new measure of productivity change is applied to investigate the productivity evolution of Tunisian banks during 1992–2014

It should be noted that the Tunisian context deserves to be studied for several reasons. First, the financial system remains excessively bank-based despite the reforms undertaken to establish a market-based financial sector. Second, external finance to Tunisian firms is provided mainly by banks. Moreover, according to numerous reports of the World Bank (2004, 2014), the International Monetary Fund (2002, 2010, 2015) and rating agencies (Fitch Ratings, 2006, 2007), the high level of nonperforming loans remains potentially a major source of vulnerability for the whole Tunisian banking system.

In fact, nonperforming loans impose costs on the economy and hinder efforts to further liberalize capital movements. Large non provisioned nonperforming loans increase the cost of bank intermediation and keep interest rates high, so that interest paying loans are penalized in order to subsidize bad loans, contributing to the weakness of the Tunisian banks and putting them in a competitive disadvantage vis-à-vis foreign banks, namely their European counterparts. Most importantly, large under provisioned provisioning nonperforming loans create a negative perception of Tunisian banking system. This is likely depriving Tunisia of better access to international capital markets, as it affects investor confidence.

Therefore, we can assert that this paper addresses a gap in banking literature by introducing, for the first time, a productivity growth decomposition specific to Tunisian banks, shedding light on the impact of nonperforming loans. Past studies have treated nonperforming loans in various ways, such as uncontrollable inputs (Drake and Hall, 2003; Hughes and Mester, 2010), a quality variable (Hughes and Mester, 1998), or undesirable outputs in the banking production process (Berg et al., 1992; Fukuyama and Weber, 2008; Park and Weber, 2006; Barros et al., 2012; Assaf et al., 2013; Glass et al., 2014; Mamatzakis et al., 2015). In line with recent literature, we categorize nonperforming loans as undesirable outputs in our productivity decomposition. Given the substantial volume of nonperforming loans in Tunisia, we anticipate that they exert an influence on bank productivity. It is plausible that banks may encounter challenges in receiving principal and interest payments on these loans, depending on the financial health of the borrowers. Consequently, these overdue loans could elevate the operating costs of banks in the short run, potentially penalizing overall bank productivity.

This study contributes to the existing literature in both theoretical and empirical dimensions. By introducing a new measure of productivity change based on the hyperbolic distance function, we diversify methodological choices for practical researches. Also, by examining productivity features of banks in Tunisia, we cover the lack in researches dealing with the Tunisian banking sector.

The remaining part of the paper is structured as follows. Section 2 provides a literature review on bank productivity; Section 3 describes the methodology; Section 4 defines bank inputs and outputs and outlines practical implementations; Section 5 discusses the findings; while the last Section presents the conclusions.

2. Literature review

This section draws attention to the literature on bank productivity, focusing on non-Parametric frontier approach (e.g. DEA) and parametric frontier approach (e.g. SFA, DFA and TFA) used to decompose total factor productivity (TFP) growth. According to Kumbhakar and Lovell (2003), both approaches require the calculation or estimation of production technology presentation to answer the following questions; whether the Tunisian banking industry has experienced productivity progress/ or regress; what is the major driver of productivity change? Nevertheless, in a stochastic environment, only the parametric approach is able to provide responses to both questions. In what follows, we present studies using both approaches to measure bank productivity.

2.1. Non-parametric studies

A structured survey of the relevant literature is reported in Table 1. As documented in the comprehensive review by Fethi and Pasiouras (2010), the dominant strand of empirical research on bank productivity prior to 2010 relies on Data Envelopment Analysis (DEA) combined with the Malmquist productivity index (Malmquist, 1953). The Malmquist Total Factor Productivity (TFP) index measures productivity change between two periods by computing the ratio of distance functions relative to a common production technology (Casu et al., 2004). When derived from an output distance function, an index value greater (less) than unity indicates productivity growth (decline) between the reference and subsequent periods.

The Malmquist framework has been widely applied in banking studies, including Berg et al. (1992), Grifell-Tatjé and Lovell (1997), Mlima (1999), Wheelock and Wilson (1999), Rebelo and Mendes (2000), Alam (2001), Mukherjee et al. (2001), Casu et al. (2004), Tortosa-Ausina et al. (2008), Fiordelisi and Molyneux (2010), and Kao and Liu (2014). A substantial body of work focuses on productivity change during periods of financial deregulation in the 1980s and 1990s. Evidence from Norway (Berg et al., 1992) and Sweden (Mlima, 1999) suggests that deregulation was initially associated with productivity declines, followed by strong post-reform recoveries, particularly among commercial banks. Similar findings are reported for Portugal, where productivity gains were largely driven by technological progress during the 1990s (Rebelo and Mendes, 2000).

For the United States, studies by Wheelock and Wilson (1999), Alam (2001), and Mukherjee et al. (2001) document the presence of technological progress alongside heterogeneous productivity outcomes across banks. While long-run productivity growth is observed, transitional periods are characterized by rising inefficiencies and uneven adoption of new technologies.

Research on emerging and transition economies further enriches the literature. Studies on Portugal (Canhoto and Dermine, 2003), Turkey (Isik and Hassan, 2003a, 2003b), and Greece (Tsionas et al., 2003) generally report productivity improvements following liberalization, though the sources of growth differ. In particular, productivity gains in Turkey are mainly attributed to efficiency improvements, whereas technological change plays a dominant role in other contexts. Financial crises, however, tend to induce temporary productivity regressions driven by technological setbacks.

Methodological refinements addressing the deterministic nature of DEA-based Malmquist indices have been proposed through bootstrapping techniques (Gilbert and Wilson, 1998; Tortosa-Ausina et al., 2008), allowing for statistical inference and confidence interval construction. More recently, Kao and Liu (2014) advocate a probabilistic version of the Malmquist index to account for externalities inherent in banking operations, providing more informative productivity assessments.

Cross-country analyses, particularly within Europe, reveal heterogeneous productivity trajectories. Casu et al. (2004) report modest productivity growth in British, French, and German banks, contrasted with stronger performance in Spanish and Italian institutions. Fiordelisi and Molyneux (2010) further demonstrate that technological change constitutes the primary channel through which productivity growth enhances shareholder value.

Finally, the traditional Malmquist index has been extended by Luenberger (1992) into the Luenberger productivity indicator, which allows for the simultaneous expansion of outputs and contraction of inputs. Applications of this indicator in banking—covering Korea, Spain, Europe, China, and India (Park and Weber, 2006; Epure et al., 2011; Williams et al., 2011; Chang et al., 2012; Fujii et al., 2014)—suggest that it offers a flexible and informative alternative for analyzing productivity dynamics in the banking sector.

Table .1: Survey of Non Parametric productivity studies in banking sector

Authors	Applied countries	Period	Methodology	Decomposition	International Comparison
Berg.al(1992)	Norway	1980-89	Malmquist (DEA)	Without decomposition	No
Mlima (1999)	Swede	1984-95	Malmquist (DEA)	Without decomposition	No
Rebelo and Mendes (2000)	Portugal	1990-97	Malmquist (DEA)	TC, EC	No
Wheelock and Wilson (1999)	US	1984-83	Malmquist (DEA)	TC, EC, SEC	No
Alam (2001)	US		Malmquist (DEA)	TC, EC	No
Mukherjee *et al.* (2001)	US	1984-90	Malmquist (DEA)	TC, EC, SEC	No
Canhoto and Dermine (2003)	Portugal		Malmquist (DEA)	TC, EC	No
Isik and Hassan (2003a)	Turkey	1981-90	Malmquist (DEA)	TC, EC, SEC	No
Isik and and Hassan (2003b)	Turkey	1992-96	Malmquist (DEA)	TC, EC, SEC	No
Grifell-Tatje and Lovell (1997)	Spain	1986-93	Malmquist (DEA) Generalized	TC, EC, SEC	No
Tsionas *et al.* (2003)	Greece	1993-98	Malmquist (DEA) Bootstrap	TC, EC	No

Gilbert and Wilson (1998)	Korea	1980-84	Malmquist (DEA) bootstrap	TC, EC, SEC	No
Tortosa-Ausina *et al.* (2008)	Spain	1992–98	Malmquist (DEA) bootstrap	TC, EC, SEC	No
Kao and Liu (2014)	Taiwan		Malmquist (DEA) stochastic	Without decomposition	No
Casu et al. (2004)	Europe	1994-2000	Malmquist (DEA)	TC, EC, SEC;	yes
Fiordelisi and Molyneux (2010)	Europe		Malmquist (DEA)	TC, EC, SEC;	yes
Park and Weber (2006)	Korea	1992-2002	Luenberger productivity (DEA)	TC, EC	No
Epure et al. (2011)	Spain	1998-2006	Luenberger productivity (DEA)	TC, EC, SEC	No
Williams et al. (2011)	Europe	1996-2003	Luenberger productivity (DEA)	TC, EC	No
Chang et al. (2012)	China	2002-2009	Luenberger productivity (DEA)	TC, EC	No
Fujii et al. (2014)	India	2004-2011	Luenberger productivity (DEA)	TC, EC	No

Efficiency Changes (EC), Technical Change (TC), Scale Efficiency Change (SEC)

2.2. Parametric studies

A survey of the relevant literature is presented in Table 2. Numerous studies have employed different econometric model specifications—namely cost, profit, and distance functions—to estimate total factor productivity (TFP) change in the banking sector. Existing research is particularly extensive for the U.S. and European banking industries (e.g., Kim and Weiss, 1989; Stiroh, 2000; Chaffai et al., 2001; Kumbhakar et al., 2001; Orea, 2002; Berger and Mester, 2003; Kumbhakar and Sarkar, 2003; Casu et al., 2004; Koutsomanoli-Filippaki et al., 2009; Feng and Serletis, 2010; Boucinha et al., 2013; Casu et al., 2013; Feng and Zhang, 2012, 2014). In what follows, we review these studies according to the econometric specifications adopted.

Studies based on cost functions constitute an important strand of the literature. Kim and Weiss (1989) estimated a system comprising a translog cost function and factor cost-share equations to examine the impact of branch expansion on TFP growth in Israeli banks over the period 1979–1982. Their results indicated an average annual TFP growth of 7.79%, with technical change contributing more than branch expansion, although both were significant drivers, particularly for small banks. Similarly, Stiroh (2000) analyzed productivity growth in U.S. bank holding companies during the 1990s using alternative cost-function-based econometric approaches. Across different specifications, the results were robust, indicating an average annual productivity growth of approximately 0.4%.

Using panel data for Indian banks from 1985 to 1996, Kumbhakar and Sarkar (2003) estimated a translog shadow cost function combined with shadow cost-share equations within a seemingly unrelated regression framework. They decomposed TFP growth into scale, technological change, and a residual component influenced by regulatory distortions. Their findings revealed a decline in productivity growth during deregulation, followed by a recovery afterward, with scale effects emerging as the primary driver across ownership types. Employing stochastic frontier methods, Boucinha et al. (2013) estimated a cost function for Portuguese banks and found that technological progress was the dominant source of TFP growth between 1992 and 2006.

Other studies have relied on profit functions. Berger and Mester (2003) used cost and profit functions to assess productivity changes in U.S. banks from

1991 to 1997, defining productivity growth as changes in best-practice technology and inefficiency. Kumbhakar et al. (2001), using a translog profit function, decomposed productivity growth for Spanish savings banks into technical change and efficiency change, identifying significant technological progress alongside substantial inefficiency. Extending this approach, Lozano-Vivas and Pasiouras (2014) incorporated off-balance-sheet activities in a parametric productivity framework applied to an international sample.

A third strand of the literature employs distance functions. Chaffai et al. (2001) used a stochastic output distance function to decompose the Malmquist productivity index for banking industries in major European countries, finding that environmental factors were more influential than pure technological change. Orea (2002) proposed a parametric decomposition of a generalized Malmquist index using a distance function and showed that TFP growth in Spanish savings banks was mainly driven by technical progress, with a positive contribution from scale effects. Koutsomanoli-Filippaki et al. (2009) parameterized a directional distance function to estimate the Luenberger productivity indicator for Central and Eastern European banks, concluding that technological change was the main source of productivity growth.

More recently, Feng and Zhang (2012, 2014) employed a true random-effects stochastic distance frontier model to account for unobserved heterogeneity among U.S. banks and measured TFP growth using the output-distance-function-based Divisia index proposed by Feng and Serletis (2010). Casu et al. (2004) compared parametric and non-parametric productivity measures for European banks, finding that productivity growth was largely driven by technical change rather than efficiency catch-up. Casu et al. (2013) further combined DEA, SFA, and meta-frontier analysis to examine productivity change in Indian banks, highlighting technology heterogeneity across ownership structures.

Finally, it is worth noting the emergence of semi-parametric approaches in the efficiency and productivity literature (Sun and Kumbhakar, 2013; Sun et al., 2015), which aim to relax functional form assumptions while retaining desirable statistical properties.

Table 2 : Survey of Parametric productivity studies in banking sector

Authors	**Applied countries**	**Period**	**Methodology**	**Decomposition**	**International Comparison**
Stiroh (2000)	US	1991-97	TFP: cost and profit functions	Without decomposition	No
Berger and Mester (2003)	US	1991-97	TFP: cost and profit functions	1)Change in best practice Change in 2)inefficiency Change in 3)business conditions	No
Lozano-Vivas and Pasiouras (2014)	84 countries	1999-2006	TFP: cost and profit functions	1)Change in best practice Change in 2)inefficiency Change in 3)business conditions	Yes
Feng and Zhang (2012)	US	1997–2006	Divisa productivity index: Parametric output distance function	1)Technical change 2)Efficiency change	No
Feng and Zhang (2014)	US	1997–2010	Divisa productivity index: Parametric output distance function	1)Technical change 2)Efficiency change	No

Chaffai et al. (2001)	France, Germany, Italy, and Spain	1993- 97	Malmquist: Parametric output distance function	1)pure technological effect and 2)environmental effect	Yes
Kumbhakar et al. (2001)	Spain	1986-95	TFP: profit function	1)Technical change 2)Efficiency change	No
Orea (2002)	Spain	(1985-1998),	Malmquist: Parametric output distance function	1) Efficiency Changes 2)Technical Change 3)Scale Effect	No
Casu et al. (2004)	Europe	1994-2000	Malmquist: Parametric output distance function	1) Scale efficiency change 2)Technical efficiency change 3) Technological change	Yes
Koutsomanoli-Filippaki et al. (2009)	Central and Eastern European countries	1998-2003	Luenberger productivity index: directional distance function	1)Technical efficiency change 2) Technological change	Yes
Boucinha et al. (2013)	Portugal	1992-2006	TFP: cost function	1) Scale efficiency change	No

				2)Technical efficiency change 3) Technological change	
Kim and Weiss (1989)	''Israeli''	1979-1982	TFP: cost function	1)scale economy and output growth, 2) branching effect, and 3) technological change effect	No
Kumbhakar and Sarkar (2003)	Indian	1985-1996	TFP: cost function	1) scale factor, 2)technological change, and 3)miscellaneous part	No
Casu et al. (2013))	Indian	1992-2009	metafrontier - Divisa and Malmquist-cost function	1) Scale efficiency change 2)Technical efficiency change 3) Technological change	No

To end with, a part from enhancing the literature concerning parametric productivity, our study aims to take into account the impact of undesirable outputs contribution on TFP change, by further exploring the main effects of non-performing loans on Tunisian banks' productivity change.

3. Methodology and Empirical Procedure

3.1. Technology

We assume that the production technology T and a set of entities (k=1…K) use a vector of N inputs vector $x \in \Re_+^N$ into M desirable output vector $y \in \Re_+^M$, and J undesirable outputs vector $b \in \Re_+^j$, by a compact production possibility set

$$T \equiv \{(x,y,b) : x \text{ can produce } (y,b)\} \tag{1}$$

which satisfies the customary axioms listed in Färe and Primont (1995). Following Cuesta et al. (2009), the hyperbolic distance function is defined by

$$D_H(x,y,b) = \min_{\delta}\{\delta > 0: (x, \frac{y}{\delta}, \delta b) \in T\} \tag{2}$$

This represents the simultaneous maximum expansion of y and contraction of b undesirable outputs that places the economic entity on the boundary of the technology T. The range of this hyperbolic distance function is $0 < D_H \leq 1$.

The hyperbolic distance function (2) treats desirable outputs and undesirable outputs asymmetrically. If $D_H(x,y,b) = 1$, the provided observation lies on the boundary of the production possibility set, where it is not possible to expand its output (*y*) or reduce its (*b*) simultaneously under the existing technology. In this context, the economic entity is defined as an efficient producer. If $D_H(x,y,b) \rangle 1$, nevertheless, this economic entity has the potential to enhance its efficiency by simultaneously increasing its output (*y)* and reducing its (*b*) compared to the efficient producer, therefore it is regarded as an inefficient producer.

The technology T exhibits a non-decreasing behaviour in the desirable output and a non-increasing pattern in the undesirable output and inputs. Additionally, it adheres to the principle of almost homogeneity.

$$D_H(x, \theta y, \theta^{-1} b) = \theta \, D_H(x,y,b), \quad \forall \theta \rangle 0 \tag{3}$$

which means that if the desirable outputs y are increased by a given proportion and the undesirable outputs b are reduced by the same proportion for a given set of inputs, then the hyperbolic distance function increases by that same proportion[1].

We can also extend the hyperbolic distance function in equation (2) and define an

$\tilde{x}$ - input saving hyperbolic distance function D_{ISH} as:

$$D_{ISH}(\tilde{x},x,y,b) = \min_{\delta}\{ \delta > 0: (\delta\tilde{x},x, \frac{y}{\delta},\delta b) \in T\} \tag{4}$$

which has properties as the hyperbolic distance function and satisfies almost homogeneity given by

$$D_{ISH}(\theta^{-1}\tilde{x},x,\theta y,\theta^{-1}b) = \theta D_{ISH}(\tilde{x},x,y,b), \quad \forall\theta\rangle 0 \tag{5}$$

The enhanced hyperbolic distance function D_{EH} further reduces all inputs. It is introduced by Cuesta et al. (2009) and also defined as:

$$D_{EH}(x,y,b) = \min_{\delta}\{ \delta > 0: (\delta x, \frac{y}{\delta},\delta b) \in T\} \tag{6}$$

which satisfies almost homogeneity given by

$$D_{EH}(\theta^{-1}x,\theta y,\theta^{-1}b) = \theta D_{EH}(x,y,b), \quad \forall\theta\rangle 0 \tag{7}$$

3.2. Translog Specification and Stochastic Frontier Approach

To assess efficiency, we employ a translog functional form for the parametric distance function, as specified by Coelli and Perelman (1999). This choice is motivated by the desirable properties of flexibility, ease of calculation, and the imposition of homogeneity.

The stochastic translog panel data specification, with N inputs, M desirable outputs and J undesirable outputs is defined as:

[1] As noted by Cuesta et al. (2009), Cuesta and Zofío's (2005) proof of the almost homogeneity property for the hyperbolic distance function ignoring undesirable outputs can be easily extended to D_H and our remaining two other distance functions.

$$\ln D_{kt}\ (x_{kt}, y_{kt}, b_{kt}) = \alpha_0 + \sum_{n=1}^{N}\alpha_n \ln x_{n,kt} + \sum_{m=1}^{M}\beta_m \ln y_{m,kt} + \sum_{j=1}^{J}\gamma_j \ln b_{j,kt}$$
$$+0.5\sum_{j=1}^{J}\sum_{j'=1}^{J}\gamma_{jj'} \ln b_{j,kt} \ln b_{j',kt} + 0.5\sum_{n=1}^{N}\sum_{n'=1}^{N}\alpha_{nn'} \ln x_{n,kt} \ln x_{n',kt} + 0.5\sum_{m=1}^{M}\sum_{m'=1}^{M}\beta_{mm'} \ln y_{m,kt} \ln y_{m',kt}$$
$$+\sum_{m=1}^{M}\sum_{n=1}^{N}\delta_{mn} \ln y_{m,kt} \ln x_{n,kt} + \sum_{n=1}^{N}\sum_{j=1}^{J}\psi_{nj} \ln x_{n,kt} \ln b_{j,kt} + \sum_{m=1}^{M}\sum_{j=1}^{J}\lambda_{mj} \ln y_{m,kt} \ln b_{j,kt} + v_{kt}$$
$$= TL\ (x_{kt}, y_{kt}, b_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt}$$
$$k = 1 \ldots K, \quad t = 1 \ldots T \qquad (8)$$

Where D_{kt} is the distance function for entitie k at time t, t is also a time variable that represents technology, and ε_{kt} is a random term distributed around zero, $v_{kt} \sim \aleph\left(0, \sigma_\varepsilon^2\right)$

Using the almost homogeneity condition in equation (3) and choosing y_M as the normalizing variable for the hyperbolic distance function in equation (8), we have

$$D_{H,kt}\ \left(x_{kt}, \frac{y_{kt}}{y_{M,kt}}, y_{M,kt} \times b_{kt}\right) = \frac{D_{H,kt}}{y_{M,kt}} \qquad (9)$$

Taking logarithm on both sides of equation (9) and combining with equation (8), we obtain

$$\ln\left(\frac{D_{H,kt}}{y_{M,kt}}\right) = TL\ (x_{kt}, y^*_{kt}, b^*_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt} \qquad (10)$$

which yield

$$-\ln\left(y_{M,kt}\right) = TL\ (x_{kt}, y^*_{kt}, b^*_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt} - \ln\left(D_{H,kt}\right) \qquad (11)$$

where TL represents the translog function, $y^*_{kt} = \dfrac{y_{kt}}{y_{M,kt}}$, $b^*_{kt} = y_{M,kt} \times b_{kt}$ and all terms involving y_M are null.

Defining $u_{kt} = \ln\left(D_{H,kt}\right)$ as an inefficiency term in the stochastic frontier analysis framework, we get the estimated econometric function as:

$$-\ln\left(y_{M,kt}\right) = TL\ (x_{kt}, y^{*}_{kt}, b^{*}_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt} - u_{kt} \qquad (12)$$

Similarly, using the almost homogeneity conditions in equations (5) and (7), we have following functions to be estimated respectively for the input saving hyperbolic distance function and the enhanced hyperbolic distance function as:

$$-\ln\left(y_{M,kt}\right) = TL\ (\tilde{x}^{*}_{kt}, x_{kt}, y^{*}_{kt}, b^{*}_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt} - u_{kt} \qquad (13)$$

$$-\ln\left(y_{M,kt}\right) = TL\ (x^{*}_{kt}, y^{*}_{kt}, b^{*}_{kt}, \alpha, \beta, \gamma, \delta, \psi, \lambda) + v_{kt} - u_{kt} \qquad (14)$$

Where $\tilde{x}^{*}_{kt} = y_{M,kt} \times w_{kt}$, $x^{*}_{kt} = y_{M,kt} \times x_{kt}$, $y^{*}_{kt} = \dfrac{y_{kt}}{y_{M,kt}}$ and $b^{*}_{kt} = y_{M,kt} \times b_{kt}$

We use stochastic frontier analysis (SFA) to estimate hyperbolic functions. This study employs a one-step model proposed by Battese and Coelli (1995). The translog HDFs to be estimated is given by:

$$\text{-ln}y_{Mkt} = TL\ (.) \underbrace{- u_{kt} + v_{kt}}_{\varepsilon_{kt}} \qquad (15)$$

Where, $\text{-ln}y_{Mkt}$ corresponds to the dependent variable and the $\varepsilon_{kt} = v_{kt} - u_{kt}$ is the composed error term. The v_{kt} are assumed to be independently and identically distributed as $\aleph\left(0, \sigma_v^2\right)$, independently distributed of the u_{kt}. The inefficiency term u_{kt} is nonnegative. the truncation point is - μ_{kt}.The truncated inefficiency term u_{kt} is independently but not identically distributed and takes the form $u_{kt} \rightarrow \aleph\left(\mu_{kt}, \sigma_u^2\right)$; $\mu_{kt} = \delta_0 + \delta z_{kt}$, where z_{kt} captures the observed bank-specific and environmental factors which explain the differences in efficiency across banks and δ is a vector of parameters to be estimated.

The efficiency of the *k*th bank is then given by:

$$Eff_{kt} = \exp\left(-u_{kt} \middle| \varepsilon_{kt}\right) = \exp\left(-\delta_0 - \delta z_{kt}\right) \qquad (16)$$

Here, we also use maximum likelihood estimation to determine values of the unknown parameters in the above model. The expressions for the likelihood function and efficiency point estimator are presented in Battese and Coelli (1993).

Overall TFP change adjusted to undesirable outputs and its decomposition

Naturally, as distance functions can be estimated parametrically, they also constitute the building blocks for the measurement of productivity change and its decomposition into basic sources of efficiency change and technical change. Though, to apprehend technological change, the function needs to include a time trend variable. Thus, (8) is extended to

$$\begin{aligned}
\ln D_H(x_{kt}, y_{kt}, b_{kt}) &= \alpha_0 + \sum_{n=1}^{N}\alpha_n \ln x_{n,kt} + \sum_{m=1}^{M}\beta_m \ln y_{m,kt} + \sum_{j=1}^{J}\gamma_j \ln b_{j,kt} \\
&+0.5\sum_{j=1}^{J}\sum_{j'=1}^{J}\gamma_{jj'} \ln b_{j,kt} \ln b_{j',kt} + 0.5\sum_{n=1}^{N}\sum_{n'=1}^{N}\alpha_{nn'} \ln x_{n,kt} \ln x_{n',kt} + 0.5\sum_{m=1}^{M}\sum_{m'=1}^{M}\beta_{mm'} \ln y_{m,kt} \ln y_{m',kt} \\
&+\sum_{m=1}^{M}\sum_{n=1}^{N}\delta_{mn} \ln y_{m,kt} \ln x_{n,kt} + \sum_{n=1}^{N}\sum_{j=1}^{J}\psi_{nj} \ln x_{n,kt} \ln b_{j,kt} + \sum_{m=1}^{M}\sum_{j=1}^{J}\lambda_{mj} \ln y_{m,kt} \ln b_{j,kt} \\
&+\pi_t t + 0.5\pi_{tt} t^2 + \sum_{n=1}^{N}\alpha_{nt}\, t \ln x_{n,kt} + \sum_{m=1}^{M}\beta_{mt}\, t \ln y_{m,kt} + \sum_{j=1}^{J}\gamma_{jt}\, t \ln b_{j,kt} + \rho \ln E_{kt} \\
&+\sum_{n=1}^{N}\rho_{xn} \ln E_{kt} \ln x_{n,kt} + \sum_{m=1}^{M}\rho_{ym} \ln E_{kt} \ln y_{m,kt} + \sum_{j=1}^{J}\rho_{jb} \ln E_{kt} \ln b_{j,kt} + v_{kt} \\
&= TL(x_{kt}, y_{kt}, b_{kt}, t, \alpha, \beta, \gamma, \delta, \psi, \lambda, \pi, \rho) + v_{kt} \\
& k = 1 \ldots K, \quad t = 1 \ldots T \qquad (17)
\end{aligned}$$

Where *E* is a quasi- fixed input; *t* is a time trend representing technological change and appears in three different forms: (i) standalone in a first and second order; (ii) cross products with inputs; and (iii) cross products with outputs (desirables and undesirables).

As equation (17) can be interpreted as a quadratic function in the variables of lny, lnb, lnx, and t, we can apply Diewert's (1976) Quadratic Identity Lemma to this distance function (refer to appendix A.5.1). Following Orea (2002), we use the identity to write the change in the hyperbolic distance function with undesirable output (17), from one period to the next for each bank k as:

$$\begin{aligned}&\ln D_H\left(x^{t+1}, y^{t+1}, b^{t+1}, t+1\right) - \ln D_H\left(x^t, y^t, b^t, t\right)\\ &\quad = \frac{1}{2}\sum_{m=1}^{M}\left(\frac{\partial \ln D_H(.,t+1)}{\partial \ln y_m} + \frac{\partial \ln D_H(.,t)}{\partial \ln y_m}\right)\left(\ln y_m^{t+1} - \ln y_m^t\right)\\ &\quad + \frac{1}{2}\sum_{n=1}^{N}\left(\frac{\partial \ln D_H(.,t+1)}{\partial \ln x_n} + \frac{\partial \ln D_H(.,t)}{\partial \ln x_n}\right)\left(\ln x_n^{t+1} - \ln x_n^t\right)\\ &\quad + \frac{1}{2}\sum_{j=1}^{J}\left(\frac{\partial \ln D_H(.,t+1)}{\partial \ln b_j} + \frac{\partial \ln D_H(.,t)}{\partial \ln b_j}\right)\left(\ln b_j^{t+1} - \ln b_j^t\right)\\ &\quad + \frac{1}{2}\left(\frac{\partial \ln D_H(.,t+1)}{\partial t} + \frac{\partial \ln D_H(.,t)}{\partial t}\right) \qquad (18)\end{aligned}$$

A logarithmic Hyperbolic Malmquist productivity index $\ln HM(.,t)$ can be defined as

$$\begin{aligned}\ln(ATFPC) = \ln HM(.,t) &= \frac{1}{2}\sum_{m=1}^{M}\left(\frac{\partial \ln D_H(.,t+1)}{\partial \ln y_m} + \frac{\partial \ln D_H(.,t)}{\partial \ln y_m}\right)\left(\ln y_m^{t+1} - \ln y_m^t\right)\\ &- \frac{1}{2}\sum_{n=1}^{N}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln x_n} + \frac{-\partial \ln D_H(.,t)}{\partial \ln x_n}\right)\left(\ln x_n^{t+1} - \ln x_n^t\right)\\ &- \frac{1}{2}\sum_{j=1}^{J}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln b_j} + \frac{-\partial \ln D_H(.,t)}{\partial \ln b_j}\right)\left(\ln b_j^{t+1} - \ln b_j^t\right) \qquad (19)\end{aligned}$$

The left-hand side of equation (3) can be interpreted as a logarithmic index of Adjusted Total Factor Productivity Change (ATFPC, adjusted for undesirable outputs). In a comprehensive sense, ATFPC is defined by subtracting the weighted average growth rates of inputs (*x*) and undesirable outputs (*b*) from that of desirable outputs (*y*). The weights used in this calculation are the distance elasticities of inputs, undesirable outputs, and desirable outputs[2], respectively. Rearranging equation (18), ln (ATFPC) can be decomposed as:

$$\begin{aligned}\ln(ATFPC) = \ln HM(.,t) &= \ln D_H(.,t+1) - \ln D_H(.,t)\\ &- \frac{1}{2}\left(\frac{\partial \ln D_H(.,t+1)}{\partial t} + \frac{\partial \ln D_H(.,t)}{\partial t}\right) \qquad (20)\end{aligned}$$

[2] Note that the distance elasticities of inputs and undesirable outputs are negative, while the distance elasticities of desirable outputs are positive.

Equation (20) parametrically decomposes the logarithmic **A**djusted **T**otal **F**actor productivity **C**hange into two meaningful parts adjusted to undesirable outputs namely, the changes in adjusted technical efficiency (indicative of catching up) and changes in adjusted technology (indicative of innovation) follow a decomposition that aligns with the non-parametric decomposition of the Hyperbolic Malmquist productivity index into efficiency change and technology change, as introduced in Chapter 2 of this thesis. As highlighted by Orea (2002), the negative sign of the second term in equation (20) transforms technical progress into a positive value and vice versa.

If we exclude the undesirable output in equation (19), we obtain the logarithmic conventional total factor productivity change (TFPC) as follows: [insert the expression for TFPC].

$$\ln(\mathrm{TFPC}) = \ln M(.,t) = \frac{1}{2}\sum_{m=1}^{M}\left(\frac{\partial \ln D_H(.,t+1)}{\partial \ln y_m} + \frac{\partial \ln D_H(.,t)}{\partial \ln y_m}\right)\left(\ln y_m^{t+1} - \ln y_m^{t}\right)$$

$$-\frac{1}{2}\sum_{n=1}^{N}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln x_n} + \frac{-\partial \ln D_H(.,t)}{\partial \ln x_n}\right)\left(\ln x_n^{t+1} - \ln x_n^{t}\right) \qquad (21)$$

Therefore, the relationship between our adjusted TFP change and the conventional TFP change is

$$\ln HM(.,t) = \ln M(.,t) - \frac{1}{2}\sum_{j=1}^{J}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln b_j} + \frac{-\partial \ln D_H(.,t)}{\partial \ln b_j}\right)\left(\ln b_j^{t+1} - \ln b_j^{t}\right) \qquad (22)$$

Equation (22) ignores the contribution of scale economies to productivity change that can only be recognized under the best practice technology allowing for various returns to scale. Therefore, productivity change and its decomposition need to be redefined under the best practice technology.

To extend the decomposition of $\ln HM(.,t)$ and to allow for the effect of various returns to scale, Orea drew on the ideas suggested by Denny et al. (1981) and developed a generalized Malmquist productivity index that can incorporate the scale effect. Using input distance elasticity shares rather than distance elasticities, a generalized Hyperbolic Malmquist productivity index can be defined as

$$\ln GHM(.,t) = \ln D_H(.,t+1) - \ln D_H(.,t) - \frac{1}{2}\left(\frac{\partial \ln D_H(.,t+1)}{\partial t} + \frac{\partial \ln D_H(.,t)}{\partial t}\right)$$

$$-\frac{1}{2}\sum_{j=1}^{J}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln b_j} + \frac{-\partial \ln D_H(.,t)}{\partial \ln b_j}\right)\left(\ln b_j^{t+1} - \ln b_j^{t}\right) \qquad (23)$$

$$+\frac{1}{2}\sum_{n=1}^{N}\left(RTS(.,t+1).e_n(.,t+1) + RTS(.,t).e_n(.,t)\right)\left(\ln x_n^{t+1} - \ln x_n^{t}\right)$$

$$= \ln HM(.,t) + \frac{1}{2}\sum_{n=1}^{N}\left(RTS(.,t+1).e_n(.,t+1) + RTS(.,t).e_n(.,t)\right)\left(\ln x_n^{t+1} - \ln x_n^{t}\right)$$

Where $e_n(.,t) = \dfrac{\partial \ln D_H(.,t)/\partial \ln x_n}{\sum_{n=1}^{N}\partial \ln D_H(.,t)/\partial \ln x_n}$ and $RTS(.,t) = \left(-\sum_{n=1}^{N}\partial \ln D_H(.,t)/\partial \ln x_n\right) - 1$

To calculate the $\ln GHM(.,t)$ and its four sources we need the estimation of the translog Hyperbolic distance function [8] by imposing almost homogeneity in outputs and making an assumption about the error structure. The estimated parameters are then used to calculate technical **Eff**iciency **C**hange (**EFFC**), **T**echnical **C**hange (**TC**), **Sc**ale **Ef**fect (**SCEF**) and **U**ndesirable **O**utputs **Ef**fect (**UOEF**). The generalized lnATFPC is the sum of these four components. The multiplicative form of the generalized ATFPC is given by (24), as follows:

$$ATFPC = GHM = EFFC \times TC \times UOEF \times SCEF \qquad (24)$$

Where, $EFFC = \exp\left(\ln D_H(.,t+1) - \ln D_H(.,t)\right) = \dfrac{D_H(.,t+1)}{D_H(.,t)} = e^{\left(-u_{t+1} + u_t\right)}$;

$$TC = \exp\left(-\frac{1}{2}\left(\frac{\partial \ln D_H(.,t+1)}{\partial t} + \frac{\partial \ln D_H(.,t)}{\partial t}\right)\right);$$

$$UOEF = \exp\left(-\frac{1}{2}\sum_{j=1}^{J}\left(\frac{-\partial \ln D_H(.,t+1)}{\partial \ln b_j} + \frac{-\partial \ln D_H(.,t)}{\partial \ln b_j}\right)\left(\ln b_j^{t+1} - \ln b_j^{t}\right)\right);$$

$$SCEF = \exp\left(\frac{1}{2}\sum_{n=1}^{N}\left(RTS(.,t+1).e_n(.,t+1) + RTS(.,t).e_n(.,t)\right)\left(\ln x_n^{t+1} - \ln x_n^{t}\right)\right)$$

Data

This study uses data from the balance sheets and income statements of individual commercial banks obtained from Tunisian bank association (TBA) for the period from 1992 to 2014. The balanced panel data, which consist of 230 observations, include 10 commercial banks. These 10 banks consist of 7 private banks and 3 state banks.

To define outputs and input prices, we follow the widely accepted intermediation approach (Sealey and Lindley, 1977). In our distance functions, in line with Fukuyama and Weber (2009), Barros et al. (2012), Assaf et al. (2011), we consider total loans and other earning assets as desirable outputs and non-performing loans as undesirable output, produced by a set of input such as, labor, physical capital and borrowed funds. Equity is included in the distance functions as a quasi-fixed input (Hughes and Mester, 2013). Table 3 describes the explanatory variables used to estimate the distance functions.

Table 3: Key variables description in our panel data

Variable	Symbol	Name	Description
Outputs	Y1 Y2 b	Total loans Other earning assets Nonperforming loans	Sum of short- and long-term loans Total earning assets less Total loans Total of classified loans [3](class2+3+4)
Inputs	X1 X2 X3	Physical capital Borrowed funds Labour	Fixed assets Total funding Personnel expenses
Other variable	X4=E	Equity	Equity capital

It should be noted that even if non-performing loans do not technically generate productive assets, non-performing loans have a direct impact on banks' provisions and contingencies and net profits. Therefore, they should be included in the banks' production process as an undesirable output (Juo, J. C. 2014). Furthermore, in an analysis of the earnings efficiency of banks, Färe et al. (2004) found that the use of bank capital as a quasi-fixed inputs is sufficient to take into account both risk-based capital requirements and the risk/return trade-

[3] Prudential regulations introduced in 1991 invited banks to classify their assets into four categories according to their delinquency. Banks are required to constitute a loss provision of 20 percent for loans in the second category (0-90 days past due), 50 percent for category three (90-180 past due), and 100 percent for the fourth category (180-360 past due).

off that bank owners face. Thus, we include equity as a quasi-fixed input in estimating the distance functions.

Table .4: Summary statistics of key variables in our panel data

Inputs and outputs	Mean	Median	Max	Min
y1 : Total loans	2 055 049	1 518 318	6 927 167	241 061
y2 : Other earning assets	489 338	327 355	2 597 095	38 568
b : Non performing	471 146	331 982	2 399 618	74 575
x1 : Labor	41 838	32 497	164 744	5 508
x2 : Borrowed funds	2 313 763	1 766 748	7 824 141	284 543
x3 : Physical capital	48 870	41 882	212 814	6 581
x4 : Equity	235 079	173 972	670 302	499

Table 4 presents descriptive statistics of all input and output variables used in this study. These statistics indicate that Borrowed funds plays the most important role in the production process of commercial Tunisian banks. These funds generate costs. A lower funding cost is one of the key factors that determine a bank's net interest margin. A bank has a funding cost advantage when it pays less interest on borrowed funds compared to other banks. This directly improves net interest margins (NIM). So, the borrowed funds are the input most requested to be saved among the inputs.

Empirical Results

All the stochastic frontier models presented above are estimated using maximum likelihood techniques based on the computer program FRONTIER 4.1 (Coelli. 1996). The maximum likelihood estimates of the three distance functions are presented in Table 5.5 To avoid convergence problems, each output and input variable except technology variable t has been divided by its geometric mean; hence the first order coefficients can be interpreted as distance elasticities evaluated at the sample mean.

Estimates of Distance Functions

Table 5 summarizes key information along with parameter estimates for our three models. Notably, the parameters γ corresponding to the estimated proportion of bank inefficiency in the composed total error term are found to be significantly different from zero in all hyperbolic distance functions.

Table 5: Maximum Likelihood Parameter Estimates for Stochastic Hyperbolic Distance Functions

Distance Function		Hyperbolic		Enhanced Hyperbolic		borrowed Funds Saving Hyperbolic	
Vbles	parm	Coef	std	Coef	std	Coef	std
Constant	α0	0.10755***	0.02183	-0.0228**	0.00992	0,04115***	0,00977
lnx1	α1	0.02913**	0.01356	0.01336	0.01127	0,02464***	0,00732
lnx2	α2	-0.87921***	0.03366	-0.49416***	0.0197	-0,48420***	0,00944
lnx3	α3	-0.04531**	0.02745	0.00925	0.01511	-0,01190	0,01415
lny2	β2	0.13197***	0.01587	0.09315***	0.01186	0,08440***	0,00797
0.5lnx1lnx1	α11	-0.09543*	0.06636	0.0107	0.04467	-0,05949*	0,03618
0.5lnx2lnx2	α22	-0.62521***	0.14172	-0.34331***	0.09871	-0,06505***	0,02212
0.5lnx3lnx3	α33	-0.18993*	0.12134	-0.19824**	0.07725	-0,04363	0,06600
lnx1lnx2	α12	0.24194***	0.07687	0.02191	0.05329	0,06599**	0,02249
lnx1lnx3	α13	-0.10397**	0.04766	-0.03813	0.03568	-0,04005*	0,02848
lnx2lnx3	α23	0.24042**	0.09934	0.24908***	0.07804	0,03977	0,03136
0.5lny2lny2	β22	0.12226***	0.02793	0.11855***	0.01695	0,04260***	0,01605
lny2lnx1	δ21	0.13055***	0.04427	-0.01408	0.02749	0,05070**	0,02216
lny2lnx2	δ22	-0.12307***	0.03703	-0.03332**	0.01633	-0,01555	0,01280
lny2lnx3	δ23	0.06008*	0.04190	0.04226**	0.01673	0,02609*	0,01526
b	γ1	-0.02469**	0.01503	-0.00459*	0.00335	0,00191	0,00702
0.5lnblnb	γ11	-0.03222***	0.01130	-0.05094***	0.01029	-0,00491	0,00588
lnblny2	λ12	-0.04578***	0.01468	-0.05112***	0.0119	-0,02604***	0,00908
lnblnx1	ψ11	-0.05371***	0.01629	0.0149**	0.00793	-0,03672***	0,01099
lnblnx2	ψ12	0.05736**	0.02494	0.03617**	0.01458	0,00536	0,00753
lnblnx3	ψ13	0.03863*	0.02503	0.01568	0.01344	0,01322	0,01143
lnbln x4	ψ14	0.01835	0.01656	-0.01255	0.01187	0,00883	0,01092
lnx4	α4	-0.03524**	0.01365	-0.00754	0.01056	-0,01458***	0,00820
0.5lnkln x4	α44	-0.05968***	0.01614	-0.025**	0.00924	-0,02183***	0,00755
lnx1ln x4	α14	-0.04830**	0.02561	0.01661	0.01602	-0,02008	0,01627
lnx2ln x4	α24	0.01280	0.03662	-0.02233	0.02584	0,00446	0,01376
lnx3ln x4	α34	-0.05667*	0.03873	0.02026	0.02561	-0,00773***	0,01466
lny2ln x4	δ24	0.07580***	0.02477	0.02052*	0.01559	0,02371**	0,01648
T	ηt	-0.00388	0.00355	-0.00202	0.00285	-0,00238**	0,00166
0.5TT	ηtt	0.00018	0.00033	0.00007	0.00032	0,00022*	0,00015
Tlnx1	ηt1	-0.00031**	0.00018	0.00006	0.00011	-0,00015*	0,00009
Tlnx2	ηt2	-0.00422**	0.00173	-0.00149	0.00127	-0,00169**	0,00086
Tlnx3	ηt3	0.00262	0.00211	0.00091	0.00115	0,00063	0,00063
Tlny2	ηty	0.00550***	0.00151	-0.00197**	0.00094	0,00139**	0,00071
Tlnb	ηtb	0.00176**	0.00087	0.00049	0.0008	0,00079**	0,00046
Constant	△0	-1.07531**	0.66705	-0.01396	0.12145	0,10646***	0,05805
ROA	△1	-0.01055***	0.00197	-0.0071***	0.00077	-0,00468***	0,00101
ROE	△2	0.43069*	0.27688	0.00513	0.08696	-0,00414***	0,00071
KLR	△3	0.01948	0.07977	-0.00783	0.02028	0,05304**	0,02915
LIQR	△4	-3.61337***	0.69339	-0.61496***	0.12774	-1,60797***	0,22995
SR	△5	-1.72941***	0.27059	-0.52944***	0.0969	-1,04771***	0,12581
C4	△6	0.58157	0.57128	0.32126**	0.13573	1,02666***	0,14180
HHI	△7	2.36764***	0.44356	-0.02424	0.14321	1,30384***	0,16545
AD	△8	1.22371***	0.22548	0.18061**	0.07242	0,65259***	0,08074
ID	△9	-0.57502***	0.12899	-0.07062**	0.03283	-0,40486***	0,05525
SZ	△10	0.09539**	0.05613	0.00382	0.01212	-0,01034***	0,00461
OWNER	△11	0.07934**	0.04534	0.00421	0.00822	-0,00553*	0,00406
GDP	△12	-0.02605**	0.00940	-0.00135	0.00225	-0,01673***	0,00461
INF	△13	-0.00366**	0.00214	-0.0003	0.00085	0,00086	0,00058
σ^2		0.01661***	0.0027	0.00808***	0.00010	0,00316***	0,00054
γ		0.96311***	0.01112	0.30661 **	0.1280	0,91328***	0,03096
LLF		350.862		500.701		519,04938	
LR test		149.322		116.925		129,47319	
$\overline{TE}$		0.936		0,970		0.973	

Notes: 1. $\sigma^2 = \sigma_u^2 + \sigma_v^2$; $\gamma = \sigma_u^2 / \left(\sigma_u^2 + \sigma_v^2\right)$; 2. χ^2 critical values for 5% significance level are in the parentheses;

3. ***. ** and * indicate 1%. 5% and 10% significance levels. respectively.

Particularly, these parameters exhibit high values, close to unity, in both the hyperbolic and financial saving hyperbolic distance functions. This indicates that a substantial portion of the variations in observed productions from the frontier can be attributed to bank inefficiency, in contrast to the Enhanced hyperbolic distance function.

The final row in Table 5 presents the results of a one-sided log-likelihood ratio (LR) test comparing the standard response function (OLS) to the full frontier model. The null hypothesis in this test is that $\gamma = 0$ versus the alternative hypothesis that $\gamma > 0$. If the null hypothesis is accepted, it could indicate that σ^2 and Δi are both zero, suggesting that inefficiency effects in the distance function are not present. In such a case, a specification with parameters that can be appropriately estimated using ordinary least squares (OLS) might be suitable (Coelli, 1996). On the contrary, if the null hypothesis is rejected, it might suggest that a standard mean response function is not an adequate representation of the data. Notably, in all three hyperbolic distance functions, the null hypothesis is rejected in favour of the stochastic frontier distance function.

Elasticities

By looking at Table 5, it is clear that in all three specifications, the coefficients of borrowed funds inputs (α2) present the expected significantly negative sign, as any increase in their values would increase distances to the frontier. In contrast, the coefficients of physical capital inputs (α1) show an unexpected positive sign, in all specifications. The coefficients of labor inputs (α3) present the expected significantly negative sign only for the hyperbolic distance function. The coefficients of other earning assets (β2) in all three specifications have the expected positive sign; either of them is significant at the 1% level, signifying that any increase in the quantity of good outputs produced (all else being equal) would result in a smaller distance to the frontier. These findings affirm that, at the sample mean, hyperbolic, Funds saving hyperbolic, and enhanced hyperbolic distance functions exhibit a non-decreasing pattern in desirable outputs.

Table 5: Monotonicity Tests at the sample mean

Variable	Elasticity	Coefficient	Standard Error	Monotonicity	
Hyperbolic distance function					
lnx1	α1	0.02913**	0.01356	non-increasing	Non-satisfied
lnx2	α2	-0.87921***	0.03366	non-increasing	satisfied
lnx3	α3	-0.04531**	0.02745	non-increasing	satisfied
lny2	β2	0.13197***	0.01587	non-decreasing	satisfied
b	γ1	-0.02469**	0.01503	non-increasing	satisfied
lnx4	α4	-0.03524**	0.01365	non-increasing	satisfied
Enhanced Hyperbolic distance function					
lnx1	α1	0.01336	0.01127	non-increasing	Non-satisfied
lnx2	α2	-0.49416***	0.0197	non-increasing	satisfied
lnx3	α3	0.00925	0.01511	non-increasing	Non-satisfied
lny2	β2	0.09315***	0.01186	non-decreasing	satisfied
b	γ1	-0.00459*	0.00335	non-increasing	satisfied
lnx4	α4	-0.00754	0.01056	non-increasing	satisfied
Funds Saving Hyperbolic distance function					
lnx1	α1	0,02464***	0,00732	non-increasing	Non-satisfied
lnx2	α2	-0,48420***	0,00944	non-increasing	satisfied
lnx3	α3	-0,0119	0,01415	non-increasing	satisfied
lny2	β2	0,08440***	0,00797	non-decreasing	satisfied
b	γ1	0,00191	0,00702	non-increasing	Non-satisfied
lnx4	α4	-0,01458***	0,0082	non-increasing	satisfied

The coefficients of undesirable output (γ1) are significantly different from zero and also have the expected negative sign in the hyperbolic distance and enhanced hyperbolic distance functions. This finding indicates that these last distance functions are non-increasing in the undesirable output at the sample mean, as required by the monotonicity condition for this output.

The selection of one specification from the three, which proves a good fit to the data set examined, is a difficult task. Because the three specifications are not nested one inside the other, the LR test has no meaning here. Alternatively, the choice of the appropriate specification could be perceived in two criteria. The first one is the monotonicity conditions; the second is the percentage of the significant parameters.

According to table 6, most of the monotonicity properties are only fulfilled by the hyperbolic distance function specification. In addition, this same specification holds the highest percentage of significant parameters. Thus, the hyperbolic specification can be considered as the most appropriate specification, which attests to a good fit with all the data examined.

Efficiency

The summary statistics of the estimated technical efficiency are presented in Table 5. Tunisian banks have an average hyperbolic, a borrowed saving hyperbolic, and an enhanced hyperbolic technical efficiency $\left(\overline{TE}\right)$ of 0.936, 0.973, and 0,970, respectively. For average hyperbolic technical efficiency of 0.936, this indicates that on average Tunisian banks could increase loan and other earning assets by 6.8% ((1/0.936)-1 = 0.068) while simultaneously reducing nonperforming loans by 6.4% (1-0.936 = 0.064) when keeping inputs unchanged and using current technology. For Financial saving hyperbolic efficiency, Tunisian banks have a higher average efficiency score, which means on average Tunisian banks could increase loan and other earning assets by 2.7% ((1/0.973)-1 = 0.0277) while simultaneously reducing nonperforming loans and financial funds input by 2.7% (1-0.973=0.027) when keeping fixed assets and labor inputs unchanged and using current technology.

When considering enhanced hyperbolic efficiency, on average Tunisian banks could increase loan and other earning assets by 3.09% ((1/0.97)-1 = 0.0309 while simultaneously reducing nonperforming loans and all inputs by 3% (1-0.97 = 0.03) when using current technology and removing technical inefficiency.

Adjusted total factor productivity change (ATFPC) over time

The ATFP change is estimated with the panel data of all 10 Tunisian banks. It should be noted that the indices begin with the year 1992 which is the base year (index level of 1.00). The ATFP change assigns numerical vales; when a value greater than one indicates positive productivity change or productivity progress, a value less than one notes productivity decline or productivity regress. Percentage change in productivity is given by (Productivity Change – 1) x 100.

The Table 7 and figure 1 report the average values (annually) of ATFP change over time. We may note that the banking sector has experienced no change in mean TFP over the sample period (1992-2014). However, in the first sub-period 1992-96, ATFP exhibited a change at an average of 0.5%, thanks to the restructuring program of the Tunisian banking system, which began in 1987 and intended to enhance the competition within the banking sector, mobilize savings and led to a more efficient allocation of resources.

Table 6 : Average Adjusted total factor productivity change over time

Year	All Banks	Public Banks	Private Banks
1992	1.000	1.000	1.000
1993	0.973	0.983	0.958
1994	1.004	1.04	0.948
1995	1.029	0.997	1.076
1996	1.014	1.064	0.939
1997	0.97	0.97	0.97
1998	0.99	0.975	1.006
1999	1.057	1.065	1.049
2000	0.956	0.974	0.939
2001	1.01	1.01	1.011
2002	0.99	0.99	0.99
2003	0.991	1.011	0.983
2004	0.991	1.024	0.977
2005	1.006	1.004	1.006
2006	0.999	0.991	1.002
2007	1.004	1.015	0.999
2008	1.009	1.013	1.007
2009	0.961	1.033	0.929
2010	1.015	1.017	1.014
2011	0.981	0.972	0.984
2012	1.064	1.027	1.081
2013	0.997	1.01	0.992
2014	0.987	0.934	1.01
1992-96	1.005	1.021	0.980
1997-01	0.997	0.999	0.995

2002-10	0.996	1.011	0.990
2011-14	1.007	0.986	1.017
1992-14	1.000	1.005	0.994

Source : Author's calculations.

Indeed, to successfully achieve this program, reforms have been carried out in connection with interest rate liberalization and credit allocation, introduction of new indirect monetary policy, strengthening prudential regulation, opening the financial sector to foreign financial institutions and promotion of the equity market. In the second sub-period 1997-01, Tunisian banks experienced productivity regress -0.3%, which could be expected since they were undergoing a high volume of NPLs and low level of provisioning. The ratio of gross NPLs to total gross claims declined from 23% in 1997 to 21% in 2000, but it remained high by international standard. Furthermore, the level of provisioning of banks' NPLs that varied between 42% and 59% over this sub-period remained low, although most banks are in compliance with current provisioning regulations[4] set by the BCT.

In the third sub-period (2002-2010), Tunisian banks recorded a productivity regress -0.4% which was more important than the one recorded in the second sub-period. It should be noted that despite the significant improvement in financial soundness indicators over the period 2002-2010, the level of nonperforming loans (NPLs) remained relatively high hampering banks profitability and development and calling for additional provisions. The higher level of NPLs is the fallout of the tourism crisis in 2002, which has not yet been resolved. In the last sub-period 2011-2014 that coincides with the post-revolution period, the ATFP exhibited a change at an average of 0.7% thanks to the establishment of an ambitious program of structural reforms proposed by the government authorities, which included as a priority, the development of the financial sector through the strengthening of banking regulation and in particular the restructuring of public banks. These reforms may be able to improve investments and create a more favorable business climate.

[4] These regulations do not require loans backed by real estate collateral to be provisioned, even though realizing real estate collateral suffers from long delays in judicial procedures.

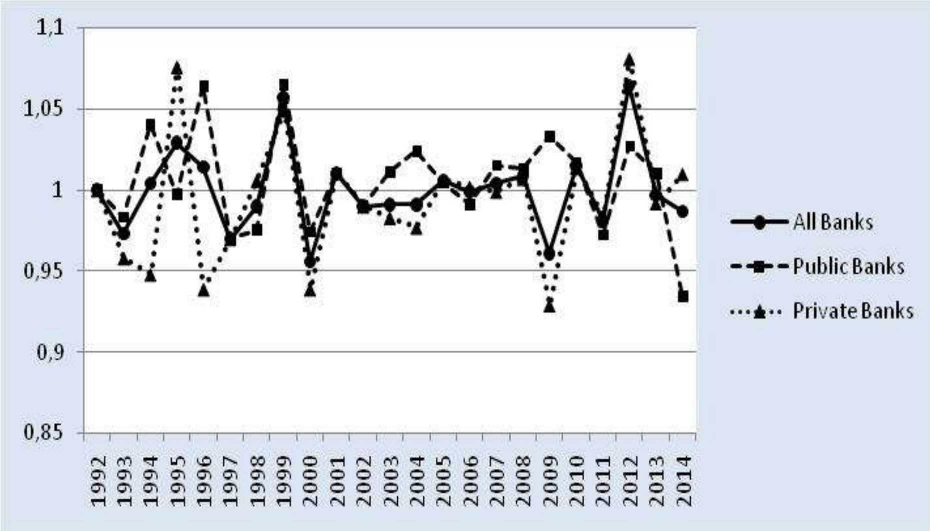

Figure 1: Average total factor productivity change over time

At the bank category level, public banks have the highest ATFP change 0.5% on average while private banks have the lowest ATFP change -0.6% on average. Although the rate of ATFP change remain strikingly different, it is widely agreed that private banks are the main contributors to poor performance. As seen in Figure 5.6.1, the TFP change of all banks follows the similar pattern as the ATFP change of private banks. In other words, the shape of the ATFP change is determined by private banks ATFP change.

Adjusted total factor productivity change decomposition

To shed more light on the contribution of each component of ATFP change, we report the average values of the effect of each component in Table 5.8, according to equation (24) of our model. We may note that over the studied period, banking sector has experienced progress in term of Efficiency change and technical change by 0.5%, and 0.3%, respectively. However, scale effect changes and NPLs effect changes have declined by -0.1% and -0.4%, respectively. Thus, over the entire period (1992-2014), there was no change in the average ATFP.

ATFP progress has been recorded in only ten out of the 23 years reported (1994, 1995, 1996, 2001, 2005, 2007, 2008 2010 and 2012) and the rest exhibited ATFP regress. The rates of ATFP change ranged between 6.4% (2012) and 0.4% (2007 and 1994) showing productivity progress. In contrast, the rates of ATFP change ranged between -4.4% (2000) and -0.1 % (2006) noting a productivity regress.

The average annual change rates showed that during the first sub-period 1992-96, ATFP change in the banking sector as a whole has improved at the rate of 0.5% due to increases in efficiency change (1.5%) which offset deterioration in the scale effect change, technical change and NPLs effect change. During the second sub-period 1997-2001, ATFP change in all banks, on average, declined at a rate of -0.3% due to large losses in scale effect change (0.3%). During the third sub-period 2002-2010, ATFP change again declined but at a greater rate of -0.4% due to large losses in efficiency changes (-0.3%) and the negative effect of NPLs (-0.6%). In the last sub-period 2011-2014, all banks recorded mean TFP progress of 0.7% which is attributed to potentials gains generated by efficiency change (0.9%), scale effect change (0.1%), and technical change (0.8%).

Table 7: Total factor productivity change decomposition over time (all banks)

	ATFPC	***EFFC***	***TC***	***SCEF***	***UOEF***	**TFPC**
Years	(1)=(2)*(3)*(4)*(5)	(2)	(3)	(4)	(5)	(6)=(2)*(3)*(4)
1992	1.000	1.000	1.000	1.000	1.000	1.000
1993	0.973	1.016	0.996	0.990	0.973	0.974
1994	1.004	1.020	0.997	0.992	0.993	1.012
1995	1.029	1.007	0.998	1.000	1.024	1.034
1996	1.014	1.017	0.999	0.998	0.996	1.028
1997	0.970	0.993	0.999	0.992	0.985	0.954
1998	0.990	0.984	1.000	0.999	1.007	0.974
1999	1.057	1.005	1.000	1.006	1.062	1.069
2000	0.956	1.039	1.000	0.987	0.950	0.981
2001	1.010	1.008	1.001	0.999	1.002	1.018
2002	0.990	0.987	1.002	1.000	1.001	0.979
2003	0.991	0.999	1.002	0.999	0.990	0.992
2004	0.991	1.001	1.003	0.997	0.990	0.993
2005	1.006	1.008	1.003	0.998	0.996	1.015
2006	0.999	1.005	1.004	1.000	0.990	1.008
2007	1.004	1.002	1.004	1.002	0.996	1.011
2008	1.009	1.007	1.005	0.998	1.000	1.019
2009	0.961	0.969	1.005	1.003	0.983	0.939
2010	1.015	1.002	1.006	1.004	1.004	1.026
2011	0.981	0.980	1.007	1.001	0.993	0.969
2012	1.064	1.051	1.008	1.002	1.003	1.130
2013	0.997	1.001	1.008	0.999	0.989	1.005
2014	0.987	1.003	1.009	1.000	0.976	0.998
1992-1996	1.005	1.015	0.998	0.995	0.996	1.012
1997-2001	0.997	1.006	1.000	0.997	1.001	0.999
2002-2010	0.996	0.997	1.003	1.000	0.994	0.998
2011-2014	1.007	1.009	1.008	1.001	0.990	1.026
1992-2014	1.000	1.005	1.003	0.999	0.996	1.006

Source: Author's calculations. ATFPC Adjusted Total Factor

Productivity Change, EFFC Efficiency Change, TC Technical Change, SCEF Scale Effect Change, UOEF Undesirable Output Effect Change and TFPC Total Factor Productivity Change

In order to further refine our analyses, we seek in the following step to know the evolution of each component over the studied period. We note that after, adding-in the additional component of the undesirable output effect (NPLs effect), TFP change declines from 0.6% to reach 0% per year. As seen in Figure 2, the NPLs effect (UOEF) of all banks follows the similar pattern as the ATFP change.

In fact, The NPLs effect (UOEF) was consistently the driving force of TFP change. Over the whole period, the NPLs explain the decrease in the TFP change, on average to -0.4%. However, the positive effect of the NPLs recorded by all banks during the second sub-period (1997-2002) attests to the success of the law launched in 1997 governing the creation of private asset management companies (AMCs) charged with the purchase and collection of NPLs. This measure enabled the NPLs ratio to fall from 23% in 1997 to 21% in 2000.

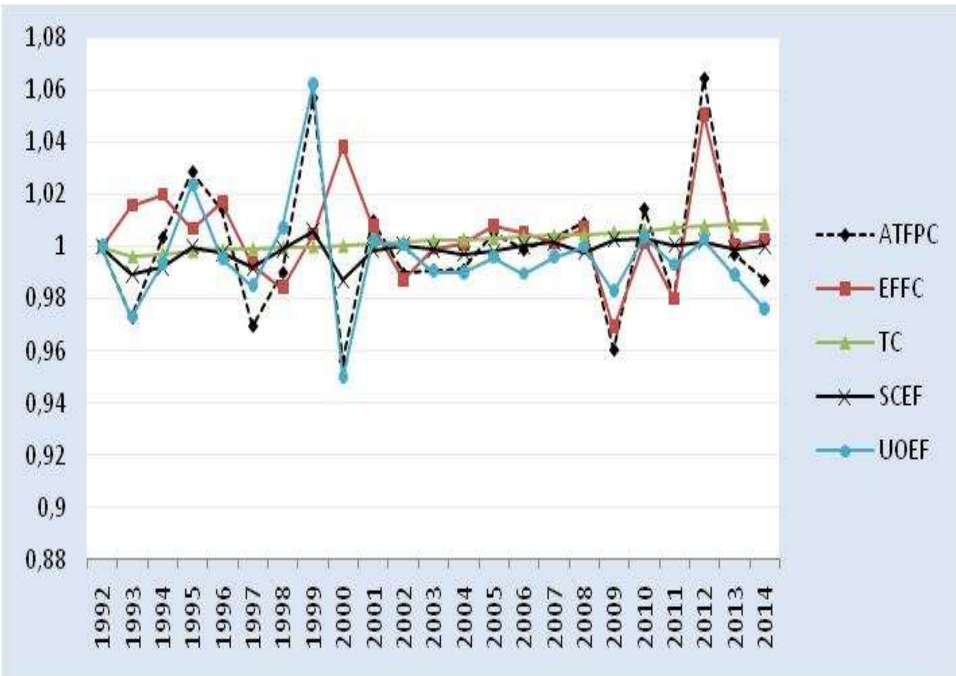

Figure 2: Average TFP change decomposition over time

It is worth noting that the negative effect of NPLs on the TFP change corroborates the results found by Altunbas et al. (2000) and Mamatzakis et al. (2015). Indeed, by incorporating the nonperforming loan ratio as a control variable in the cost function, Altunbas et al. (2000) identified a positive relationship between nonperforming loans and inefficiency. Similarly, Mamatzakis et al. (2015) reported a negative impact of problem loans on Japanese bank performance.

Regarding the technical change, we find strong evidence of technological progress over the whole studied period. Indeed, technology accounts for the increase in ATFP change on average at 0.3%. As seen in Figure 2, the pattern of technical change shows a positive upward trend. It is from 1999 that technological progress has a positive effect on the TFP change. This might be due to a program of modernization of the banking system implemented under the leadership of the BCT. This program aims primarily to set up an infrastructure for cash clearing between banks, expand the use of credit cards, strengthen the information technology safety of banks and modernize the training bank staff. Since October 2002, banks have been required to replicate all electronic information on an independent backup server.

Turning to the scale effect change, we find the negative impact of scale over the whole studied period. This result is consistent with the expectation that the Tunisian banks are operating with decreasing economies of scale (see chapter 3 of the present thesis). Therefore, the scale effect contributes around -0.1% on average to TFP change. Specifically, the negative contributions of scale to ATFP change in the first (1992-1996) sub-period and the second (1997-2001) one were -0.5 % and -0.3%, respectively. However, the third (2002-2010) sub-period and the fourth (2011-2014) sub-period exhibit slight positive scale effects, which are 0.0% and 0.1% respectively. These effects are not enough to offset the negative ones. Our findings coincide with the view suggesting that the wave of restructuring-spanning from the privatization to the Merger- is not sufficient to internalize the negative scale effect.

Figure 2 provides a clear image that technical efficiency change is very imperative to explain the APTF change. Indeed, the efficiency change is very volatile over the sample period and the whole pattern of ATFP change has been highly influenced. So, the efficiency change contributes around 0.5% on average to TFP change. This could be explained by reforms being carried out in connection with interest rate liberalization and credit allocation, introduction of new indirect monetary policy, strengthening of prudential regulation, opening of the financial sector to foreign financial institutions and promotion of the equity market.

Adjusted total factor productivity change decomposition respecting banks ownership status

At this level of analysis, we have thus far assumed that private and public banks originated from the same legal and business environment. However, it might be questionable to aggregate private and public banks into a common frontier. In this phase, we test whether there is a significant difference between the technology employed by private and public banks. Public and private banks might have different objectives that are not closely aligned. Generally, the principal goal of the government is to try to maximize social welfare. Therefore, public banks might be seen as vehicles for raising capital to finance projects with high social returns, but possibly low profit returns. In contrast, private banks are more incentive to maximize profits or to minimize costs over a longer term in order to survive. Finally, public banks seem to endure serious agency problems when compared to private banks.

Following the procedures outlined in Elyasiani and Mehdian (1992), as well as Isik and Hassan (2002) and others, we utilize a set of parametric tests (t-test and sd-test) and non-parametric tests (Kolmogorov–Smirnov and Mann–Whitney [Wilcoxon Rank-Sum]) to examine the null hypothesis of identical frontiers between the efficiency and productivity of private and public banks.

Table 8: Summary of parametric and non- parametric tests for the null hypothesis that public (pub) and private (priv) banks possess identical technologies

Productivity measures(b)	**Test groups(a)**							
	Parametric test				**Non-Parametric test**			
Individual test s	t- test		sd-test		Mann–Whitney [Wilcoxon Rank-Sum test		Kolmogorov–Smirnov [K–S] test	
null hypotheses H0	Mean priv = Mean pub		$\frac{\sigma(priv)}{\sigma(pub)} = 1$		Median priv = Median pub		Dist.private = Dist.public	
Test statistics	T (prb >t)	decision (d)	F (prb < F)	decision (d)	Z (prb > z)	decision (d)	k-s (prb >k-s)	decision (d)
ATFP	-1.045 (0.301)	Acc H0	1.6382 (0.266)	Acc H0	-1.315 (0.1885)	Acc H0	0.2727 (0.387)	Acc H0
EFFC	-0.096 (0.923)	Acc H0	1.6069 (0.285)	Acc H0	-0.211 (0.8326)	Acc H0	0.1818 (0.860)	Acc H0
TC	-2.197 (0.033)	Rej H0	0.8313 (0.676)	Acc H0	-1.989 (0.0467)	Rej H0	0.2727 (0.387)	Acc H0
SCEF	-0.297 (0.767)	Acc H0	2.6242 (0.032)	Rej H0	0.095 (0.9246)	Acc H0	0.1818 (0.860)	Acc H0
UOEF	-0.625 (0.535)	Acc H0	3.7900 (0.003)	Rej H0	-1.973 (0.0485)	Rej H0	0.3636 (0.109)	Acc H0

Source: *Author's calculations. (a) The null hypothesis that public (pub) and private (priv) banks are drawn from the same productivity population. The numbers in parentheses are the p-values associated with the relative test.(b) ATFPC Adjusted Total Factor Productivity Change, EFFC Efficiency Change, TC Technical Change, SCEF Scale Effect Change, Undesirable Output Effect Change and TFPC Total Factor Productivity Change. (d)* Acc H0: Accept H0, Rej H0: Reject H0

However, in table 9, we do not find statistically significant difference in the mean between the efficiency and productivity of public and private banks. The t-test results are further confirmed by the results derived from the non-parametric Mann–Whitney [Wilcoxon Rank-Sum] and Kolmogorov–Smirnov tests. Hence, we conclude that there is no significant statistical difference between the efficiency and productivity of public and private banks.

Based on the previous tests, we failed to reject the null hypothesis at the 5% levels of significance indicating that the public and private banks are drawn from the same population and have identical technologies. Furthermore, the results from the sdtest for equality of variances do not reject the null hypothesis suggesting that the variances among the private owned and public owned banks are equal. Thus, we can assume that the variances among the private and public banks are equal.

In order to identify the significance of individual relationships between the decomposed components and the ATFPC index, the Pearson and Spearman rank order correlation coefficients are presented in Table 10. The Pearson correlation results confirmed that there were positive and statistically significant relationships (at 5% and 10% levels) between the ATFPC index and EFFC, SCEF and UOEF components. For all banks, the Pearson correlation coefficient between UOEF and ATFPC is 0.772, representing a high order relationship when compared to those associated with EFFC and SCEF. Furthermore, the same analysis was repeated for private banks and public ones. In the case of private banks, the highest Pearson coefficient is estimated between UOEF and ATFPC (0.817), while in the case of public banks, the highest Pearson coefficient is estimated between EFFC and ATFPC (0.712). Added to that, the Spearman rank-order correlation coefficients are all statistically different from zero indicating a strong association between the productivity index and the decomposed components.

All Spearman rank-order correlation coefficients present positive relationships between the decomposed components and the ATFPC index. The correlation coefficients suggest that all components except TC have an impact on the ATFPC index of all banks. UOEF has the greatest negative effect on the productivity change of private banks, whereas EFFC has the strongest positive effect on the productivity change of public banks.

Table 9 : Pearson (p) and Spearman(s) Correlation Coefficients among ATFPC index and Decomposed Components of all, private and public Banks

All banks

	ATFPC	EFFC	TC	SCEF	UOEF
ATFPC					
P	1				
S	1				
EFFC					
P	0.473**	1			
S	0.420**	1			
TC					
P	0.0211	-0.254	1		
S	0.100	-0.121	1		
SCEF					
P	0.353*	-0.347	0.514**	1	
S	0.558**	-0.374*	0.526**	1	
UOEF					
P	0.772**	0.0464	-0.073	0.482**	1
S	0.779**	-0.131	-0.102	0.672**	1

Private banks

	ATFPC	EFFC	TC	SCEF	UOEF
ATFPC					
P	1				
S	1				
EFFC					
P	0.456**	1			
S	0.289	1			
TC					

P	0.313	0.099	1		
S	0.2612	0.0620	1		
SCEF					
P	0.6194**	-0.116	0.558**	1	
S	0.672**	-0.396*	0.472**	1	
UOEF					
P	0.817**	0.0453	0.147	0.702**	1
S	0.724**	-0.371*	-0.0055	0.815**	1

Public banks

	ATFPC	EFFC	TC	SCEF	UOEF
ATFPC					
P	1				
S	1				
EFFC					
P	0.712**	1			
S	0.733**	1			
TC					
P	|0.0099	-0.2571	1		
S	-0.1289	-0.2870	1		
SCEF					
P	0.539**	0.105	0.261	1	
S	0.448**	0.0371	0.4060	1	
UOEF					
P	0.487**	-0.0658	-0.151	0.271	1
S	0.627**	-0.0129	-0.1602	0.3549	1

Pearson correlation coefficient- first row of each cell. Spearman rank order correlation coefficient presented in second row of each cell.

*** Correlation is significant at the 0.05 level (2-tailed).*

** Correlation is significant at the 0.1 level (2-tailed).*

*** Spearman Correlation is significant at the 0.05 level (2-tailed).*

Adjusted total factor productivity change decomposition per banks

The average results for ATFP change and its components for all banks are presented in Table 11. The results show that only four banks have recorded an average ATFP progress over the sample period. The highest average progress in ATFP has been recorded by Bank 1 (2.4%) followed by bank 3 (1.1%), bank 5(0.3%) and Bank 8 (0.6%). The average regress in ATFP is observed in Bank

6 (-0.2%), Bank 4 (-0.9%), Bank 2 (-1.2%), Bank 7 (-0.1%), Bank 9(-0.8%) and Bank 10 (-3.7%).

Table 10: Adjusted Total Factor Productivity Change decomposition per type of banks.

Banks	*NAME*	ATFPC (1)=(2)*(3)*(4)*(5)	*EFFC* (2)	*TC* (3)	*SCEF* (4)	*UOEF* (5)
bank1	*BNA*	**1,024**	**1,013**	**1,006**	**1,001**	**1,004**
bank2	*STB*	**0,988**	**0,996**	**1,005**	**0,998**	**0,990**
bank3	*BH*	**1,011**	**1,003**	**1,003**	**1,000**	**1,005**
bank4	*UIB*	**0,991**	**1,008**	**1,003**	**0,994**	**0,987**
bank5	*ATTIJARI*	**1,003**	**1,003**	**1,003**	**0,998**	**1,000**
bank 6	*UBCI*	**0,998**	**1,000**	**1,001**	**0,998**	**0,999**
bank 7	*BIAT*	**0,999**	**1,001**	**1,004**	**0,997**	**0,998**
bank 8	*BT*	**1,006**	**1,004**	**1,001**	**1,002**	**1,000**
bank 9	*AB*	**0,992**	**1,000**	**1,002**	**1,004**	**0,986**
bank 10	*ATB*	**0,963**	**1,000**	**0,999**	**0,993**	**0,971**

Source: *Author's calculations. ATFPC Adjusted Total Factor Productivity Change, EC Efficiency Change, TC Technical Change, SCEF Scale Effect Change and UOEF Undesirable Output Effect Change.*

The decomposition of ATFP change reveals that the largest improvement in mean ATFP change for Bank1, Bank 3, Bank 5, and Bank 8 is attributed to efficiency progress, technical progress and positive effect of undesirable output. A rather contrasted picture is observed for the Bank 2, Bank 6 and Bank 10 where the efficiency regress and the negative effect of undesirable output are the principal contributor to mean ATFP regress. On the other hand, scale and undesirable outputs have the worst effect on ATFP change in Bank 7 (by -0.3% and -0.2%, correspondingly) and in Bank 4 (by -0.6% and - 1.3%, correspondingly). Significantly, mean TFP regress (-0.8%) for the Bank 9 is attributed to negative effect of undesirable outputs (-1.4%), which offset its technical progress, and positive scale effect (0.2% and 0.4%, respectively).

Four out of the 10 banks have shown mean productivity progress over the sample period. The highest mean ATFP change has been recorded by Bank1 (2.4%) followed by bank3 (1.1%), Bank8 (0.6%) and Bank5 (0.3%). All other banks have recorded mean ATFP regress ranging from -0.1% (Bank 7) to -3.7% (Bank10).

Conclusions

This study quantifies the impact of undesirable outputs on productivity change of Tunisian banks. We adopt a parametric methodology which allows a decomposition of TFP change with respect to the impact of undesirable outputs, namely Nonperforming loans. Our findings report no change in mean TFP over the studied period, due to progress in efficiency change (0.5%) and technical change (0.3%) that offset the regression in scale effect change (-0.1%) and NPLs effect change (-0.4%).

The annual average change rates show that during the first sub-period 1992-96, the ATFP change in the banking sector has improved at the rate of 0.5% due to increases in efficiency change (1.5%) which offset deterioration in the scale effect change, technical change and NPLs effect change. During the second sub-period 1997-2001, the ATFP change, on average, is found to have declined at a rate of -0.3% due to large losses in scale effect change (0.3%). During the third sub-period 2002-2010, the ATFP change declined again but at a greater rate of -0.4% due to large losses in efficiency changes (-0.3%) and negative effects of NPLs (-0.6%). In the last sub-period 2011-2014, all banks recorded mean TFP progress of 0.7% which is attributed to potential gains generated by efficiency change (0.9%), scale effect change (0.1%), and technical change (0.8%).

The estimates of productivity change for different banking ownership types reveal that public banks recorded mean TFP progress of 0.50% whereas private banks recorded regress. Public banks are found to have been more successful than the private banks in capturing benefits from changes in technology and efficiency. In addition, the changes of scale effect and undesirable output effect are found to be problematic for the private and public banks indicating that they do not operate at an optimal scale and do not efficiently manage their risk. More precisely, NPLs had the greatest negative effect on the productivity change of private banks, whereas EFFC had strongest positive effects on the productivity change of public ones.

This study provides several policy implications. Our results indicated that banks with a riskier portfolio involving a higher level of NPLs can diminish the productivity change of the Tunisian banking system as a whole. This being so, regulators must reasonably monitor and manage the level of risk in commercial banks, as well as their lending processes. Alternatively, our findings indicate that severe regulatory procedures should be implemented to reduce banks' default risk and improve their productivity.

The findings showed also that banks have experienced negative scale effect change mainly due to diseconomies of scale (i.e., decreasing returns to scale (DRS)), i.e., to being at more than the optimum size. In order to achieve optimal size, decision-makers can use the Merger and Acquisition strategy to take advantage of economies of scale by reducing costs and maximizing economic benefits. This strategy also increases the credit creation capacity of the merged or acquired bank.

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